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On the Lateral Deviation of Spherical Projectiles.

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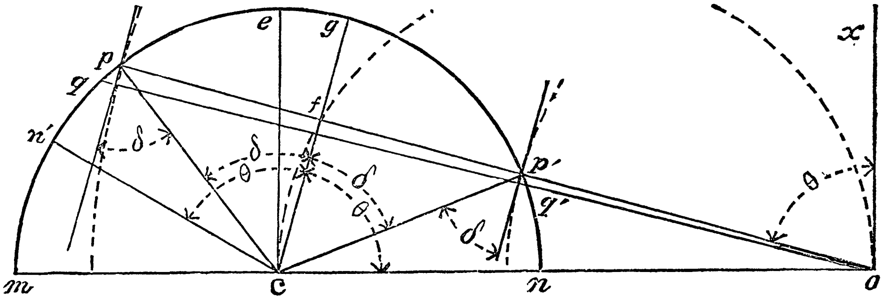
IT may be premised that the phenomenon discussed in this paper is actually observed in pitching base ball; and though it has been often asserted by non-experts that it is impossible to pitch "a curved ball," nevertheless such is not the fact, as appears from careful experiment as well as from the following theoretical investigation. The base ball is selected for experiment in preference to other projectiles, because it is possible to gain more exact information with respect to its initial twist than can be gained as to round shot for example or other spherical projectile.

Since the precise law of resistance which such a projectile experiences in its passage through the air is unknown, we shall be obliged to content ourselves with showing the direction of the deviation without being able at present to compute its numerical amount.



In the figure let c be the center of a spherical projectile whose radius is a, and let *men* be the great circle of the sphere which lies in a horizontal plane. Let us disregard the vertical component of the motion of the projectile; and let c have a horizontal motion of translation, at the instant under consideration, towards e. Also let the projectile have a motion of rotation about a vertical axis through c in a right-handed direction, *i. e.* from m to e. The motions of translation and rotation, whatever be their relative velocities, can be combined, as is well known, into a single motion of rotation about an instantaneous axis parallel to the vertical axis of rotation through c. This instantaneous axis must intersect the diameter *mn*, which is perpendicular to the direction of translation *ce*, at some point, as *o*. Let the instantaneous axis through *o* be called the axis of *z*. Also, let the distance *oc* be designated by the letter *b*.

Let *r* be the shortest (*i. e.* horizontal) distance of any element *dS* of the surface of the sphere from the axis *z*. Now pass any vertical plane through the axis of *z*, cutting the sphere in a circle whose horizontal projection is *pp'*, and similarly pass a second plane *qq'*, making an infinitesimal angle $d\theta = poq$ with *pp'*; draw *cfg* perpendicular to *pp'*, and let $\delta = fcp$, $fco = xop = \theta$,

$$\therefore cf = a \cos \delta = b \cos \theta . \quad \ldots \ldots \ldots \quad (1)$$

In the vertical circle *pp'* cut out by the first plane, and having *fp* for its radius, let ϕ be the angle between the radius *fp* and the radius drawn to any element *dS* of the surface situated on the circumference of the vertical circle *pp'*. Then r, θ, ϕ are coordinates of *dS*, but since the surface is a sphere, we obtain the following relation between these coordinates and known quantities:

$$r = b \sin \theta + a \sin \delta \cos \phi . \quad \ldots \ldots \ldots \quad (2)$$

Again, since *z* is the instantaneous axis, the motion of any element *dS* of the sphere is horizontal and perpendicular to the instantaneous radius *r* of that element, and, therefore, the relative velocities of different elements are proportional to their respective instantaneous radii *r*,

$$\therefore v = cr, \quad \ldots \ldots \ldots \quad (3)$$

in which *v* is the velocity of any element of the sphere and *c* is a constant.

Let *dS* be the quadrilateral element of the spherical surface included between the two planes *pp'*, *qq'*, making an angle $d\theta$ with each other, and two meridian planes intersecting in the line *cg* and making an angle $d\phi$ with each other. Then is *dS* ultimately a rectangle, of which the length along the meridian circle is $r \operatorname{cosec} \delta \, d\theta$, and the width along the vertical circle is $a \sin \delta \, d\phi$;

$$\therefore dS = ar \, d\theta \, d\phi . \quad \ldots \ldots \ldots \quad (4)$$

Disregarding friction, the resultant pressure *dP* on the element *dS*, as it moves through the atmosphere, is towards *c*, and is proportional to v^n (in which the exponent *n* lies between 1 and 2, but its precise value is unknown), and is also proportional to the cross section $\cos \delta \, dS$ of the stream of air which *dS* meets in its motion. Then if *c'* is some constant

$$dP = c'v^n dS = ac'c^n r^{n+1} \cos \delta \, d\theta \, d\phi . \quad \ldots \ldots \quad (5)$$

Now resolve dP into two rectangular components; the first, $\sin\delta\, dP$ acting in the plane pp' towards f, the second, $\cos\delta\, dP$ acting perpendicular to the plane pp'.

The first component $\sin\delta\, dP$ has a horizontal component of magnitude $\sin\delta\cos\phi\, dP$, which has a component parallel to co and acting from c toward o, whose magnitude is $\sin\delta\sin\theta\cos\phi\, dP$. The second component $\cos\delta\, dP$ is horizontal, and has a component of magnitude $-\cos\delta\cos\theta\, dP$ acting from c toward o. These are the only components of the normal pressure dP acting along co;

$$\therefore dX = (\sin\delta\sin\theta\cos\phi - \cos\delta\cos\theta)\, dP = \cos\psi\, dP. \quad . \quad . \quad (6)$$

is the horizontal deviating force acting on the element dS, in which ψ is the arc of the great circle joining dS to n', a point so situated in the horizontal plane that the angle $ncn' = 2\theta$. For, let dS be situated at one angle of a spherical triangle of which the remaining two are g and n'; then, since δ and θ are two of its sides and ϕ is the included angle, and ψ is the side opposite ϕ, we have

$$\cos\psi = \sin\delta\sin\theta\cos\phi - \cos\delta\cos\phi, \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\therefore dX = bc'c^n r^{n+1}\cos\theta\cos\psi\, d\theta\, d\phi. \quad . \quad . \quad . \quad . \quad . \quad (8)$$

It is readily shown that the deviating force, acting on any elementary ring of the forward half of the sphere included between pp' and qq', is from c toward o, for this deviating force is twice that obtained by integrating (8) with respect to ϕ from $\phi = 0°$ to $\phi = 180°$. And it is possible to show that the value of this integral is a positive quantity, without effecting the integration, by showing that the sum of the positive elements of the integral exceeds the sum of its negative elements. Now it is evident from (2) that, while θ is constant, r decreases as ϕ increases from 0° to 180°, but r never becomes negative in case the axis z lies without the sphere, as, in practice, it does. Again, it appears from the interpretation given to ψ that $\cos\psi$ decreases as ϕ increases from 0° to 180°, but that, so long as $\theta < 90°$, more than half the elements dS along this ring between pp' and qq' are within 90° of n', and hence the largest positive value of $\cos\psi$ numerically exceeds its largest negative value.

Therefore, in integrating (8) with respect to the independent variable ϕ from 0° to 180° (for any value of θ up to 90°), the positive elements of the integral are not only larger but more numerous than its negative elements. And if we afterwards integrate with respect to the independent variable θ from $\theta = \cos^{-1}\frac{a}{b}$ to $\theta = 90°$ (which includes the hemisphere now under consideration), the total deviating force X will act from c towards o, for each one of its elements will be positive. Furthermore, let us consider the pressures acting upon the remaining hemisphere. These pressures are less than if the projectile stood still, there being a partial vacuum behind it. An experimental comparison of the pressures on the front and back sides of moving bodies shows that the reduction of pressure on the back side below the mean never exceeds about one-half of the increase of pressure on the front side above the mean. Hence, by comparing the pressures on pairs of rings making equal angles with the vertical plane mn, it is seen that, although the deviating force caused by the pressures on the back hemisphere acts from o towards c, it does not numerically exceed about one-half of the deviating force caused by the pressures on the front hemisphere, and acting from c to o. Therefore the total deviating force caused by the normal pressures is from c toward o.

The effect of friction between the air and projectile remains now to be considered. If the air exerted equal pressures at the opposite extremities of each diameter, the friction could diminish the rotary motion, but could cause no deviation. The pressures are not, however, thus distributed. We may state the case roughly thus: in the quarter of the sphere projected on the paper in mec the average pressures are greater than in either other quarter, the average pressures in ecn are next largest; and, if e' is the other extremity of a diameter through e, the average pressures in nce' are next largest, and those in $e'cm$ are the smallest.

The difference of the pressures in the opposite quarters mce, nce' causes a difference of frictions; the same difference exists between the opposite quarters ecn, $e'cm$; and these two effects do not differ greatly in magnitude. Hence the total effect might be replaced by a friction at a point of the surface not far from e. It is evident that the effect of applying friction at such a point would be to cause the projectile to roll away from o, so that the component of the deviating force furnished by the friction is from o to c. But the amount of this force is inconsiderable compared with that caused by the differences of the normal pressures, being dependent, however, upon the roughness of the surface of the projectile. Rankine* states that Smeaton's experiments show that the coefficient of friction for the best sails of wind-mills is probably about 0.016.

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* Manual of the Steam Engine, etc. 7th ed. London, 1874, chap. VIII.